



IB MYP YEAR 5  
ASSESSMENT TASK

# Patterns in Probability

Subject:	Y10 <i>Extended</i> Mathematics	Name : (Class)	Sharon Lau (Y10T)
Topic:	Patterns in Probability		
Date of assessment:	Thursday 1 <sup>st</sup> December (session 2)		

- This task assesses Criteria B and C
- Time allowed – *one hour*
- Write your answers on the file paper provided. GDCs are allowed.

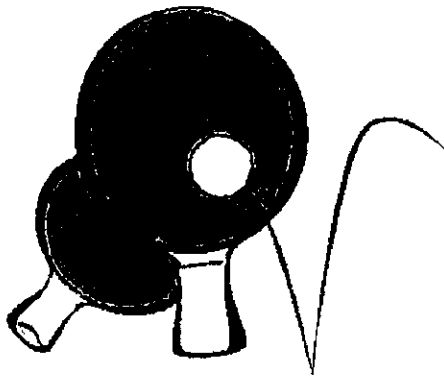
## ADVICE:

Read the criteria descriptors and task-specific clarifications carefully before you start your work. This will give you a clear understanding of what is required and what a high quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest levels in this task.

Criterion B		
Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	You are able to answer the early questions, and order your answers in a way that reveals patterns.	The student <b>applies, with some guidance</b> , mathematical problem-solving techniques to recognize simple patterns.
3-4	You develop appropriate systematic methods in order to answer the questions. The results you get help you to suggest a mathematical rule using a, b and N	The student <b>applies</b> mathematical problem-solving techniques to recognize patterns, <b>and suggests</b> relationships or general rules.
5-6	You continue with the questions, and use questions 6 and 7 as a check on your findings.	The student <b>selects and applies</b> mathematical problem-solving techniques to recognize patterns, <b>describes</b> them as relationships or general rules, and <b>draws conclusions</b> consistent with findings.
7-8	You are able to justify or prove your answer to question 6.	The student <b>selects and applies</b> mathematical problem-solving techniques to recognize patterns, <b>describes</b> them as relationships or general rules, <b>draws the correct conclusions</b> consistent with findings, and <b>provides justifications or a proof</b> .

Criterion C		
Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	The narrative is basic. Mathematical symbols are used, perhaps with <b>some</b> errors or inconsistencies.	The student shows <b>basic use</b> of mathematical language and/or forms of mathematical representation. The lines of reasoning are <b>difficult to follow</b> .
3-4	The narrative is reasonably easy to follow. Mathematical language is used in a generally accurate way. Mathematical notation is used with few errors.	The student shows <b>sufficient use</b> of mathematical language and forms of mathematical representation. The lines of reasoning are <b>clear though not always logical or complete</b> . The student moves between different forms of representation with <b>some success</b> .
5-6	The narrative is easy to follow. Mathematical arguments are presented logically. Mathematical vocabulary and notation are used accurately and appropriately.	The student shows <b>good use</b> of mathematical language and forms of mathematical representation. The lines of reasoning are <b>concise, logical and complete</b> . The student moves <b>effectively</b> between different forms of representation.

# Patterns in Probability



I play table tennis against a friend. The probability I win a point is  $a$  and the probability she wins a point is  $b$ . These probabilities stay constant throughout the game.

**(Note:  $a \neq b$ )**

***A game is won only when a player wins two consecutive points.***

*So, for example, I might win a game where the point rallies go: WLWLWLWLWW (that is, I win the game because I won the final 2 points).*

***In the following, simplify your answers wherever possible.***

1. Write a simple relationship between  $a$  and  $b$ .
2. What is the probability I win the first two points (and so win the game)?
3. What is the probability the game lasts for 3 points and I win?
4. What is the probability the game lasts for 4 points and I win?

*(You may like to repeat question 4 for when the game lasts for 5 points, or 6, or 7 and so on.)*

5. Describe mathematically any patterns you find in these probabilities.
6. What is the probability, in terms of  $a$  and  $b$ , that the game lasts for  $N$  points and I win?
7. If  $a = 0.6$ , show that the probability I win a game in 5 points or less is approximately 0.625.
8. Write a ***proof*** or ***justification*** of your answer to question 6.



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1<sup>st</sup> December, 11

1. The points  $a$  &  $b$  will determine the competitors win or lose. The points will keep counting until the game end.

$$\begin{aligned}
 & 2. P(\text{two points}) \rightarrow 2) P(2 \text{ points}) \\
 & = \cancel{P(\frac{1}{2} \times \frac{1}{2})} P(W, W_2) + P(W_2, W_1) P(\frac{1}{2} \times \frac{1}{2}) \\
 & = P(\frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2}) \\
 & = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 & 3. P(3 \text{ points}) \rightarrow 3) P(3 \text{ points}) \\
 & P(L, W, W_2) + P(W_2, W, W_1) \\
 & P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \\
 & = P(\frac{1}{8}) + P(\frac{1}{8}) \\
 & = P(\frac{2}{8}) = P(\frac{1}{4})
 \end{aligned}$$

$$\begin{aligned}
 & 4. P(4 \text{ points}) \\
 & P(LLWW) + P(WLWW) \\
 & = P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \\
 & = P(\frac{1}{16}) + P(\frac{1}{16}) \\
 & = P(\frac{2}{16}) = P(\frac{1}{8})
 \end{aligned}$$

$$\begin{aligned}
 & P(5 \text{ points}) \\
 & P(LWLWW) + P(WLLWW) + P(LLLWW) \\
 & = P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \\
 & = P(\frac{1}{32}) + P(\frac{1}{32}) + P(\frac{1}{32}) \\
 & = P(\frac{3}{32})
 \end{aligned}$$

P (6 points)

~~P(LWLLWW)~~

$$= P(LWLLWW) + P(\overset{WLLLWW}{\cancel{LWLLWW}}) + P(WLWLWW) + P(LLLLWW)$$

$$+ \cancel{P(LWLLWW)} = P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$$

$$+ P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$$

$$= P(\frac{1}{64}) + P(\frac{1}{64}) + P(\frac{1}{64}) + P(\frac{1}{64})$$

$$= P(\frac{4}{64}) = P(\frac{1}{16})$$

P (7 points)

$$= P(LWLWLWW) + P(WLLLLWW) + P(WLWLLWW) + P(LLLLLWW)$$

$$= P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$$

$$+ P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) + P(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$$

$$= P(\frac{1}{128}) + P(\frac{1}{128}) + P(\frac{1}{128}) + P(\frac{1}{128})$$

$$= P(\frac{4}{128}) = P(\frac{1}{32})$$

5)	2 points	$= \frac{1}{4} = \frac{1}{2^2}$
	3 points	$= \frac{1}{8} = \frac{1}{2^3}$
	4 points	$= \frac{1}{8} = \frac{1}{2^3}$
	5 points	$= \frac{3}{32} = \frac{3}{2^5}$
	6 points	$= \frac{1}{16} = \frac{1}{2^4}$
	7 points	$= \frac{1}{32} = \frac{1}{2^5}$
	N points	$= \frac{1}{n} = \frac{1}{2^n}$

$$6) \left(\frac{1}{2}\right)^n \times \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$$

the probability is  $\frac{1}{2^n}$

$$\frac{n}{2} \times \frac{n}{2} = \frac{1}{2^n}$$

$$\frac{2^n}{4} = \frac{1}{2^n}$$

$$2^n \times 2^n = 1$$

$$2^n \times 2^n = \frac{1}{4}$$

$$2(n \times 1^n) = \frac{1}{4}$$

$$n \times 1^n = \frac{1}{8}$$

$$\cancel{n} = \frac{1}{8} \quad n = \frac{1}{2^3}$$



7)  $\frac{1}{n} = \frac{6}{10}$   
 $\frac{1}{n} \times 10 = 6$   
 $\frac{10}{n} = 6$   
 $10 = 6n$   
 $\frac{10}{6} = n$   
 $\frac{5}{3} = n$   
 $n = \frac{5}{3}$

$\Rightarrow 0.6 = \frac{1}{2^n}$   
 $\frac{6}{10} = \frac{1}{2^n}$   
 $\frac{3}{5} = \frac{1}{2^n}$

Probability of 5 points =  $\frac{3}{32}$   
 $= 0.09375$   
 $n = \frac{5}{3} = 1.6666$   
 $0.625 = \frac{5}{8}$

8 In Q. 3 the probability of 3 points is  $\frac{1}{8}$   
and now I ~~can~~ calculate n points probability  
is  $\frac{1}{8}$  so I think n points is equal 3

[illegible]